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Math Research

Euler's Phi Function

Euler's phi function, represented by φ(n), is defined as the number of integers between 1 and n (excluding n) that are relatively prime. Relatively prime numbers are the integers between 1 and a number that are not factors or multiples of the factor. Note that 1 is relatively prime. In φ(n), where n is a prime number, the answer can be found by subtracting one from n. For example, if n=3, the relatively prime numbers are 1 and 2, resulting in 2 numbers. When n=5, the relatively prime numbers are 1,2,3, and 4. The realization that the answer is n-1 came from the fact that if you were to list the numbers from 1 to any prime number, the only non-relatively prime number is the number itself. In the case where n is the square of a prime number, φ(n) is equal to . The relatively prime numbers of 4 are 1 and 3, totaling 2. The relatively prime numbers of 9 are 1,2,4,5,7, and 8, totaling 6. At that point, it was realized that the numbers that were not relatively prime was the square root of n and its multiples. This always totaled the . The third case is when n is the product of two primes, p and q. The answer to φ(n) is (p-1)(q-1). The relative primes of 6 are 1 and 5, totaling 2. The relative primes of 10 are 1,3,7, and 9, totaling 4. It was realized that 6 was the product of 2 and 3. The φ(2) and the φ(3) are 1 and 2 respectively. 1\*2=2 In addition, it was realized that the factors of 10 are 5 and 2. The φ(5) and the φ(2) are 4 and 1. 4\*1=4. It seems the φ(p)\* φ(q) always yields the answer for φ(n) where n is the product of two prime numbers. It was established before that the φ of a prime number is equivalent to the prime number minus 1. Through substitution, φ(n) where n is the product of two prime numbers, p and q, can be simplified to (p-1)(q-1).